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Gauge dependence of the on-shell renormalized mixing matrices

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Abstract

It was recently pointed out that the on-shell renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the method by Denner and Sack causes a gauge parameter dependence of the amplitudes. We analyze the gauge dependence of the on-shell renormalization of the mixing matrices both for fermions and scalars in general cases, at the one-loop level. We then show that this gauge dependence can be avoided by fixing the counterterms for the mixing matrices in terms of the off-diagonal wave function corrections for fermions and scalars after a rearrangement, in a similar manner to the pinch technique for gauge bosons. We finally present explicit calculation of the gauge dependence for two cases: the CKM matrix in the Standard Model, and left-right mixing of scalar quarks in the minimal supersymmetric standard model.

1 Introduction

Particles in the same representation under unbroken symmetries can mix with each other. The neutral gauge bosons, quarks, and massive neutrinos in the Standard Model (SM) are well-known examples. New particles in extensions of the Standard Model also show the mixings. For example, in the minimal supersymmetric (SUSY) standard model (MSSM) [1], a very promising extension, superpartners of most SM particles show the mixing [1, 2]. The mixing of particles is expressed in terms of the mixing matrix, which represents the relations between the gauge eigenstates and the mass eigenstates of the particles. The mixing matrices always appear at the couplings of these particles in the mass eigenbasis.

Because of the fact that mass eigenstates at the tree-level mix with each other by radiative corrections, the mixing matrices have to be renormalized [3, 4] to obtain ultraviolet (UV) finite amplitudes. Denner and Sack have proposed [4] a simple scheme to renormalize the mixing matrix of Dirac fermions at the one-loop level, which is usually called the on-shell renormalization scheme. They have required the counterterm for the renormalized mixing matrix to completely absorb the anti-hermitian part of the wave function correction δZ_{ij} for the external on-shell fields. This definition works very well for the subtraction of the ultraviolet divergence and dependence on the renormalization scale. The renormalization procedure is universal for any processes with the particles as external states. It also absorbs the $\mathcal{O}(1/(m_i^2 - m_j^2))$ terms which are singular for the case $m_i \simeq m_j$. The on-shell scheme was also applied to the mixing of other fields, such as Majorana fermions [5] and complex scalar particles [6].

However, it has recently been pointed out [7, 8, 9] that in the on-shell scheme of Ref. [4] the counterterms for the Cabibbo-Kobayashi-Maskawa (CKM) matrix [10] is dependent on the gauge fixing parameter and that, as a consequence, the amplitudes of charged current interactions of quarks are also gauge dependent in this scheme. This fact motivated these authors to introduce other ways for the UV finite renormalization of the CKM matrix [7, 9]. However, their method cannot be directly applied to mixings of other particles.

In this paper we study the gauge parameter dependence of the on-shell renormalized mixing matrices in general cases. We demonstrate that this gauge dependence is a general feature for the on-shell mixing matrices. Nevertheless, at the one-loop level the on-shell mixing matrices by Ref. [4] can be modified to be gauge independent by the following procedure. First, we split the gauge-dependent parts of the wave function corrections in the similar way to the "pinch technique" [11, 12, 13]. They are then rearranged into the corresponding vertex corrections in the amplitudes and cancelled. Next, we give the counterterm for the on-shell renormalized mixing matrices in terms of the remaining, gauge-independent part of δZ_{ij}. The subtraction of the UV divergence and of the $\mathcal{O}(1/(m_i^2 - m_j^2))$ singularity is not affected by this modification. This method can be applied in a similar manner both for mixings of fermions and of scalars.

This paper is organized as follows. In section 2 we review the one-loop on-shell renormalization of the mixing matrices for scalars and fermions in general case. In section 3 their gauge dependences are analyzed by using the Nielsen identities [14, 15, 16] for self energies of scalars and fermions. We then show that the gauge dependences of the off-diagonal wave function corrections and, in consequence, of the on-shell mixing matrices can be split by the rearrangement of the loop corrections. Sections 4 and 5 present two explicit calculations of the gauge dependence of mixing matrices; CKM matrix of quarks in the SM and left-right mixing of scalar quarks (squarks) in the MSSM. Section 6 gives our conclusion.

2 On-shell renormalization of mixing matrices

Let ψ_α (with index α) be fields in gauge eigenstates, either real or complex scalars, or chiral components of Dirac or Majorana fermions. The fields in common representation under unbroken symmetries may mix with each other to form mass eigenstates. The relation between gauge eigenstates ψ_α and tree-level mass eigenstates f_i with masses m_i is expressed by an unitary matrix U as

$$f_i = U_{i\alpha}\psi_\alpha, \quad \psi_\alpha = U^*_{i\alpha} f_i. \tag{1}$$

The mixing matrix U is determined such that the tree-level mass matrix for f_i is diagonal. The couplings of f_i are always multiplied by U. For example, an amplitude $\mathcal{M}_i$ with one incoming external f_i is expressed as

$$\mathcal{M}_i = \sum_\alpha \mathcal{M}_\alpha U^*_{i\alpha}, \tag{2}$$

where $\mathcal{M}_\alpha$ has no U dependence. U is therefore very important parameter for f_i. Note that, when f are fermions, the mixing matrices U^L and U^R for chiral components f_L and f_R, respectively, are generally different from each other.

By radiative corrections, the wave functions of f_i should be renormalized. The on-shell renormalized fields f_i are related to the unrenormalized $f_i^{(0)}$ by, at the one-loop level

$$f_i^{(0)} = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}\right) f_j. \tag{3}$$

The off-diagonal parts of $\delta Z_{ij} (i \neq j)$ represent the mixing between f_i and f_j. For the relation (1) is modified as

$$\psi_\alpha = U^{(0)*}_{i\alpha} f_i^{(0)} = U^{(0)*}_{i\alpha}\left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}\right) f_j, \tag{4}$$

the wave function correction to the amplitude (2) is expressed as the replacement of U by

$$U^*_{i\alpha} \to U^{(0)*}_{j\alpha}\left(\delta_{ji} + \frac{1}{2}\delta Z_{ji}\right). \tag{5}$$

This correction is universal in any processes involving on-shell external f_i.

The explicit form of δZ_{ij} for $i \neq j$ is given in terms of the off-diagonal, flavor-mixing parts of the self energy[1] of the fields f. For scalars with unrenormalized, dimensionally regularized self energy $\Pi_{ij}(p^2)$, we have

$$\frac{1}{2}\delta Z_{ij} = \frac{1}{m_i^2 - m_j^2}\Pi_{ij}(m_j^2). \tag{6}$$

For Dirac fermions with self energy

$$\Sigma_{ij}(p) = \Sigma_{Lij}(p^2)\not{p}P_L + \Sigma_{Rij}(p^2)\not{p}P_R + \Sigma_{DLij}(p^2)P_L + \Sigma_{DRij}(p^2)P_R\ , \tag{7}$$
$$\Sigma^*_{Lji}(p^2) = \Sigma_{Lij}(p^2),\ \Sigma^*_{Rji}(p^2) = \Sigma_{Rij}(p^2),\ \Sigma_{DRij}(p^2) = \Sigma^*_{DLji}(p^2), \tag{8}$$

the corrections to chiral components of the wave functions (f_{iL}, f_{iR}) are [17]

$$\begin{aligned}\frac{1}{2}\delta Z^L_{ij} &= \frac{1}{m_i^2 - m_j^2}\left[m_j^2\Sigma_{Lij}(m_j^2) + m_i m_j\Sigma_{Rij}(m_j^2) + m_i\Sigma_{DLij}(m_j^2) + m_j\Sigma_{DRij}(m_j^2)\right]\ ,\\ \frac{1}{2}\delta Z^R_{ij} &= \frac{1}{m_i^2 - m_j^2}\left[m_i m_j\Sigma_{Lij}(m_j^2) + m_j^2\Sigma_{Rij}(m_j^2) + m_j\Sigma_{DLij}(m_j^2) + m_i\Sigma_{DRij}(m_j^2)\right]\ ,\end{aligned} \tag{9}$$

respectively. Both of Eqs. (6,9) have the factor $1/(m_i^2 - m_j^2)$ which is unique for the off-diagonal wave function corrections. These δZ_{ij} are UV divergent and depend on the gauge fixing parameters ξ for the massive gauge bosons. Note also that δZ_{ij} superficially diverge when the masses (m_i, m_j) of f_i and f_j, respectively, become close to each other. For the case of Majorana fermions [5], the self energy (7) obeys additional conditions

$$\Sigma_{Lij}(p^2) = \Sigma^*_{Rij}(p^2),\ \Sigma_{DLij}(p^2) = \Sigma_{DLji}(p^2),\ \Sigma_{DRij}(p^2) = \Sigma_{DRji}(p^2). \tag{10}$$

The condition for the wave function corrections, $\delta Z^L_{ij} = \delta Z^{R*}_{ij}$, which is necessary for keeping Majorana condition $U^L = U^{R*}$ after renormalization, then follows from Eqs. (9, 10). All subsequent discussions in this and the next sections remain unchanged by the conditions (10).

For the cancellation of the UV divergence of off-diagonal δZ_{ij} in Eq. (5), the mixing matrix U has to be renormalized [3, 4]. Assume that the renormalized U is related to the bare $U^{(0)}$ by

$$U^{(0)}_{i\alpha} = (\delta_{ij} + \delta u_{ij})U_{j\alpha}\,. \tag{11}$$

Since both $U^{(0)}$ and U are unitary, the counterterm δu should be anti-hermite. The correction factor (5) is then rewritten as

$$U^{(0)*}_{j\alpha}\left(\delta_{ji} + \frac{1}{2}\delta Z_{ji}\right) = U^*_{j\alpha}\left(\delta_{ji} + \frac{1}{2}\delta Z_{ji} - \delta u_{ji}\right). \tag{12}$$

The UV divergent part of δu is determined [4] such as to cancel that of the anti-hermitian part of δZ. For fermions, also the UV divergence of the diagonal CP-violating part

$$\frac{i}{2}\mathrm{Im}(\delta Z^L_{ii}) = -\frac{i}{2}\mathrm{Im}(\delta Z^R_{ii}) = \frac{i}{2m_i}\mathrm{Im}(\Sigma_{DLii}(m_i^2)), \tag{13}$$

[1] We assume that the absorptive part of the self energy is negligible. For its correct inclusion one has to treat f's as unstable intermediate states.

in the convention[2] which is valid both for Dirac and Majorana fermions, has to be cancelled by $\delta u_{ii}^{L,R}$. The earlier UV divergence of δu is consistent with the running of the mass matrix of f in the gauge eigenbasis [18, 19, 5]. The renormalized mixing matrix U is then given by specifying the finite part of δu.

The modified minimal subtraction ($\overline{\rm MS}$) scheme is simplest and proven to give gauge-independent renormalized parameters [20]. However, the decoupling of heavy particles is not manifest in this scheme. The cancellation of the dependence on the renormalization scale Q between running parameters and different parts of the amplitude is often quite delicate and complicated. In addition, the $\mathcal{O}(1/(m_i^2 - m_j^2))$ singularity for $m_i \simeq m_j$ remains in the amplitudes. These properties make the $\overline{\rm MS}$ scheme inconvenient in realistic studies. On the other hand, the renormalized mixing matrices may also be defined directly in terms of the physical observables. This method is manifestly independent of the gauge fixing and renormalization scale. However, the form of the counterterm strongly depends on the chosen observables and is often very complicated.

It is therefore natural to investigate the method to define the renormalized mixing matrices which are independent of the renormalization scale and of the specific processes. In the study of the radiative correction to the CKM matrix, Denner and Sack [4] proposed to cancel the total anti-hermitian part of δZ_{ij} by δu, choosing

$$\delta u_{ij} = \frac{1}{4}(\delta Z_{ij} - \delta Z_{ji}^*). \tag{14}$$

This is usually called the on-shell renormalization of the mixing matrix. Equation (5) is then rewritten as

$$U_{j\alpha}^{(0)*}\left(\delta_{ji} + \frac{1}{2}\delta Z_{ji}\right) = (U^{\rm OS})_{j\alpha}^*\left(\delta_{ji} + \frac{1}{4}(\delta Z_{ji} + \delta Z_{ij}^*)\right). \tag{15}$$

One important feature of Eq. (15) is that all $\mathcal{O}(1/(m_i^2 - m_j^2))$ singularities in δZ_{ij} are absorbed into the renormalized $U^{\rm OS}$. Also, $U^{\rm OS}$ is independent of the $\overline{\rm MS}$ renormalization scale. These properties are equally valid both for fermions and scalars.

The mixing of quarks in different generations needs a special care for there is no unique "gauge eigenbasis" for them. Instead, one can discuss only the difference between the mixing of left-handed up-type quarks and that of down-type quarks, namely the CKM matrix $V_{ij} = (U^{u_L})_{i\alpha}(U^{d_L})_{j\alpha}^*$. The counterterm for the on-shell CKM matrix is then given by [4, 7]

$$\delta V_{ij} = \delta u_{ik}^{u_L} V_{kj} + \delta u_{jk}^{d_L *} V_{ik}, \tag{16}$$

where δu^{q_L} are given by Eq. (14).

[2] For Dirac fermions, one may make the shift $(\delta Z_{ii}^L,\, \delta Z_{ii}^R) \to (\delta Z_{ii}^L + i\theta_i,\, \delta Z_{ii}^R + i\theta_i)$ by an arbitrary imaginary number $i\theta_i$. This is equivalent to the phase rotation $(f_{iL},\, f_{iR}) \to (e^{i\theta_i} f_{iL},\, e^{i\theta_i} f_{iR})$ in Eq. (3). This freedom is killed by Majorana condition. See Ref. [5] for details.

3 Gauge dependence of wave function corrections and on-shell mixing matrices

Since the proposal in Ref. [4], however, the dependence of the on-shell mixing matrix on the gauge fixing parameters ξ has not been examined for a long time. Recent studies [7, 8, 9] showed that the on-shell renormalization of the CKM matrix introduces gauge dependence into one-loop amplitudes for the $W^+ \to u_i \bar{d}_j$ decays through the counterterm δV_{ij}. They proposed alternative definitions for quark mixing matrix which are independent of the renormalization scale. References [7, 8] used a modified process-independent definition for the CKM matrix. As shown in this section, their definition strongly relies on the gauge representation of quarks. Reference [9] fixed the renormalized CKM matrix by using the amplitudes of the decays $W^+ \to u_i \bar{d}_j$ (or $t \to W^+ d_j$). To keep the renormalized CKM matrix unitary, four processes have to be selected out of nine possible ones. As a result, the forms of the corrected amplitudes become very asymmetric with respect to generation indices (i, j). Thus, both methods cannot be directly applied for the renormalization of other mixing matrices. In this section we show another way to avoid the problem of gauge dependence of the on-shell scheme of Ref. [4].

We first investigate the gauge parameter dependences of the wave function correction δZ and of the counterterm δu for the on-shell mixing matrix in general cases. We use the fact that, in the R_ξ gauge, the dependence of the one-particle irreducible Green functions on the gauge parameters ξ is controlled by the Nielsen identities [14, 15], a kind of the Slavnov-Taylor identities which follow from the extended Becchi-Rouet-Stora (BRS) symmetry [15] of the theory. The identity for the gauge parameter dependence of the inverse propagator $\Gamma_{ij}(p)$ for the transition $f_j \to f_i$ takes the following form [16]

$$\partial_\xi \Gamma_{ij}(p) \equiv \partial \Gamma_{ij}(p)/\partial \xi = -\Gamma_{\chi \bar{f}_i K_l}(p) \Gamma_{lj}(p) - \Gamma_{il}(p) \Gamma_{\chi K_{\bar{l}} f_j}(p). \tag{17}$$

Here $\Gamma_{\chi \bar{f}_i K_l}(p)$ is the vertex function with $\bar{f}_i$, χ, the "BRS variation" of the gauge parameter ξ [15, 16], and K_l, the source associated with the BRS variation of f_l. $\Gamma_{\chi K_{\bar{l}} f_j}(p)$ is its conjugate. Since the identity (17) is determined by the form of the gauge-fixing terms [16], it holds for general gauge theories in the R_ξ gauge fixing. In Eq. (17) f's are assumed to be physical fields with gauge-independent masses, not the would-be Nambu-Goldstone (NG) bosons, Fadeev-Popov ghosts, or longitudinal modes of gauge bosons. Under this condition $\Gamma_{\chi \bar{f}_i K_l}(p)$ has no tree level contribution. It is also required that the renormalization does not introduce additional gauge dependence [16]. Especially, the shift of the vacuum expectation values (VEVs) of Higgs bosons by tadpole graphs should be cancelled in a gauge-independent way.

The gauge dependence of the one-loop two-point functions $\Sigma_{ij}(p)$ of fermions is, in the tree-level mass basis, derived from general result (17) as

$$\partial_\xi \Sigma_{ij}(p) = \Lambda_{ij}(p)(\not{p} - m_j) + (\not{p} - m_i)\overline{\Lambda}_{ij}(p), \tag{18}$$

where $\Lambda(p)$ and $\bar{\Lambda}(p)$ are some one-loop Dirac spinors. After the decomposition

$$\Lambda_{ij}(p) = \Lambda_{Lij}(p^2)\not{p} P_L + \Lambda_{Rij}(p^2)\not{p} P_R + \Lambda_{DLij}(p^2) P_L + \Lambda_{DRij}(p^2) P_R, \tag{19}$$

and similar one for $\overline{\Lambda}$, the ξ dependence of the components of Σ in Eq. (7) is [16]

$$\begin{aligned}
\partial_\xi \Sigma_{Lij} &= -m_j \Lambda_{Lij} - m_i \overline{\Lambda}_{Lij} + \Lambda_{DRij} + \overline{\Lambda}_{DLij}, \\
\partial_\xi \Sigma_{Rij} &= -m_j \Lambda_{Rij} - m_i \overline{\Lambda}_{Rij} + \Lambda_{DLij} + \overline{\Lambda}_{DRij}, \\
\partial_\xi \Sigma_{DLij} &= p^2 \Lambda_{Rij} + p^2 \overline{\Lambda}_{Lij} - m_j \Lambda_{DLij} - m_i \overline{\Lambda}_{DLij}, \\
\partial_\xi \Sigma_{DRij} &= p^2 \Lambda_{Lij} + p^2 \overline{\Lambda}_{Rij} - m_j \Lambda_{DRij} - m_i \overline{\Lambda}_{DRij}.
\end{aligned} \tag{20}$$

The relations

$$\overline{\Lambda}_{Lij} = \Lambda^*_{Lji}, \;\; \overline{\Lambda}_{Rij} = \Lambda^*_{Rji}, \;\; \overline{\Lambda}_{DLij} = \Lambda^*_{DRji}, \;\; \overline{\Lambda}_{DRij} = \Lambda^*_{DLji}, \tag{21}$$

follow from the hermiticity of the effective action.

By substituting them into Eq. (9), we obtain [7] for $i \neq j$

$$\frac{1}{2}\partial_\xi(\delta Z^L_{ij}) = -m_j \overline{\Lambda}_{Rij}(m_j^2) - \overline{\Lambda}_{DLij}(m_j^2), \tag{22}$$

and similar result for δZ^R_{ij}. As a result, the original definition of the on-shell renormalized fermion mixing matrices in Eq. (14) has gauge parameter dependence. Explicit calculation shows that the gauge dependence of the counterterm δu^L_{ij} is equal to

$$\frac{1}{4}\partial_\xi(\delta Z^L_{ij} - \delta Z^{L*}_{ji}) = \frac{1}{2}\left[-m_j \overline{\Lambda}_{Rij}(m_j^2) - \overline{\Lambda}_{DLij}(m_j^2) + m_i \overline{\Lambda}^*_{Rji}(m_i^2) + \overline{\Lambda}^*_{DLji}(m_i^2)\right], \tag{23}$$

which does not vanish in general. This is also the case for δu^R_{ij} and δu_{ii}.

A remarkable fact in Eq. (22) is that the factor $1/(m_i^2 - m_j^2)$, which characterizes the off-diagonal δZ_{ij}, is cancelled for the gauge dependence. This is expected from the gauge independence of the total amplitudes [21] with gauge-independent renormalization of the couplings. Since the gauge dependence of Eq. (22) has to be cancelled by that from other parts of the amplitudes which do not have the factor $1/(m_i^2 - m_j^2)$, the factor cannot remain in Eq. (22). Similar cancellation occurs in the gauge dependence of the diagonal part $\delta u^L_{ii} = -\delta u^R_{ii}$, which is equal to

$$\frac{i}{2}\partial_\xi(\mathrm{Im}\,\delta Z^L_{ii}) = \frac{i}{2}\mathrm{Im}\left[-m_i \overline{\Lambda}_{Rii}(m_i^2) + m_i \overline{\Lambda}_{Lii}(m_i^2) + \overline{\Lambda}_{DRii}(m_i^2) - \overline{\Lambda}_{DLii}(m_i^2)\right]. \tag{24}$$

The factor $1/m_i$ in Eq. (13), which characterizes $\mathrm{Im}(\delta Z_{ii})$, is cancelled in Eq. (24). Another important point is that Eqs. (23, 24) are UV finite.

The mixing matrices of the scalars can be analyzed in the similar way. The one-loop two-point function $\Pi_{ij}(p^2)$ for scalars in the tree-level mass basis obeys the relation [16]

$$\partial_\xi \Pi_{ij}(p^2) = \Lambda_{ij}(p^2)(p^2 - m_j^2) + (p^2 - m_i^2)\Lambda^*_{ji}(p^2), \tag{25}$$

from the Nielsen identity. We assume that there are no mixings with unphysical modes. By substitution we obtain for $i \neq j$

$$\frac{1}{2}\partial_\xi(\delta Z_{ij}) = -\Lambda^*_{ji}(m_j^2). \tag{26}$$

The gauge dependence of the counterterm (14) for the on-shell mixing matrix for scalars is therefore

$$\partial_\xi(\delta u_{ij}) = -\frac{1}{2}\left[\Lambda^*_{ji}(m_j^2) - \Lambda_{ij}(m_i^2)\right], \tag{27}$$

which is UV finite but does not cancel in general. However, the factor $1/(m_i^2 - m_j^2)$ is again cancelled in Eq. (27).

According to the earlier observation, we can define the gauge-independent one-loop on-shell mixing matrices for fermions and scalars as follows. First, we split gauge-dependent parts without the factor $1/(m_i^2 - m_j^2)$ from δZ_{ij} and regard them as parts of the corrections to the attached vertex. They are eventually cancelled by the gauge dependence of the vertex and other corrections. Second, we give the counterterms for mixing matrices in terms of the remaining, gauge-independent part of δZ_{ij}. This procedure gives the one-loop corrected amplitudes which are expressed in terms of the on-shell mixing matrices and manifestly gauge independent. Of course, the choice of the gauge-independent parts of δZ_{ij} has arbitrariness. For example, we can regard the results in the R_ξ gauge with a given ξ as the gauge-independent parts.

Here we propose a method to specify the gauge-invariant parts of δZ_{ij}, inspired by the pinch technique [11, 12, 13] to define gauge-independent form factors for gauge bosons. We consider a general process with the external on-shell particle f_j which is either a fermion or a scalar, with incoming momentum p. One source of the gauge dependence of δZ_{ij} is the graph of Fig. 1(a). As pointed out in Refs. [11, 12], the longitudinal part of the propagator of the (massive) gauge boson A triggers the Ward identity at the vertex μ as

$$\not{k} = -(\not{p} - m_i) + (\not{k} + \not{p} - M) + (M - m_i), \tag{28}$$

for fermions, or

$$k^\mu(k+2p)_\mu = -(p^2 - m_i^2) + ((k+p)^2 - M^2) + (M^2 - m_i^2), \tag{29}$$

for scalars, respectively. The first two terms of Eqs. (28, 29) cancel the propagators of f_i and of the intermediate particle F with a mass M, respectively, and yield the contributions of Figs. 1(b, c) (pinching). The last terms of Eqs. (28, 29) are the effect of the spontaneous breaking of the gauge symmetry for A and are proportional to the couplings to the associated NG boson. The part of Fig. 1(a) where the last terms are picked up at the vertex μ is further decomposed into three parts by the Ward identity (28, 29) at ν. The part which cancels f_j propagator vanishes in on-shell amplitudes, while that which cancels F propagator is included in the type of Fig. 1(c). The remaining part where the last terms of the Ward identity are picked up at both vertices does not fit into Figs. 1(b, c). To satisfy the Nielsen identities (18, 25), this part has to be combined with the contribution from Fig. 1(d) by the NG boson ϕ_A to yield a gauge-independent sum. This result should be thus equal to the contribution of Fig. 1(d) in the $\xi = 1$ gauge.

The contribution of Fig. 1(b) is manifestly consistent with the Nielsen identity. In contrast, the remaining gauge-dependent part, Fig. 1(c), cannot satisfy the identity by itself because of its p independence. This part has to be cancelled by the contributions from the Higgs VEV shift [Fig. 1(e)] by the loops of unphysical modes for A and, in the case of scalars, by the "seagull" contributions with four-point couplings $f_i^* f_j A^\mu A_\mu$ [the same topology as Fig. 1(c)] and $f_i^* f_j \phi_A \phi_A$. Again, the result should be gauge-independent and therefore equal to the one in the $\xi = 1$ gauge. We have verified that, for the cases discussed in Sections 4 and 5, the earlier cancellation of the gauge dependence really occurs and that the contribution of Fig. 1(b) is equal to the difference from the result in the $\xi = 1$ gauge.

It is then natural to identify the contribution of Fig. 1(b) to δZ_{ij} as the gauge-dependent pinch term, in analogy to Ref. [12], and to regard this as a part of the vertex corrections. Then, in this manner, we may regard the on-shell mixing matrices in the $\xi = 1$ gauge as the gauge-independent ones. The cancellation of the UV divergence, renormalization scale dependence, and the $\mathcal{O}(1/(m_i^2 - m_j^2))$ singularity is not affected by this modification of the original definition of the on-shell mixing matrices. Note that the agreement of the $\xi = 1$ and the pinch technique results has been observed for the QCD correction to the off-shell quark propagator [13]. Note also that we have not considered, for scalars, the possible trigger of the Ward identity (29) at the vertex μ by the momentum $(k + 2p)_\nu$ at the vertex ν in Fig. 1(a), which was done for the couplings of the gauge and NG bosons [12] to satisfy the Ward identities among corrected vertices.

We finally comment on other definitions for the UV finite and process-independent mixing matrices for fermions. As the first example, Ref. [22] proposed a definition of the UV finite and momentum-dependent effective mixing matrices $[\overline{U}^L(p^2), \overline{U}^R(p^2)]$ for fermions. The counterterms for $\overline{U}$ are given by, instead of Eq. (9),

$$\begin{aligned} \delta \bar{u}_{ij}^L(p^2) &= \frac{1}{m_i^2 - m_j^2} \left[\frac{1}{2}(m_i^2 + m_j^2)\Sigma_{Lij}(p^2) \right. \\ &\left. \quad + m_i m_j \Sigma_{Rij}(p^2) + m_i \Sigma_{DLij}(p^2) + m_j \Sigma_{DRij}(p^2) \right] , \end{aligned} \tag{30}$$

and similar form for $\delta \bar{u}_{ij}^R(p^2)$. Similar to the on-shell U by Ref. [4], $\overline{U}(p^2)$ absorb the $\mathcal{O}(1/(m_i^2 - m_j^2))$ singularity when the couplings of f_i are expressed in terms of $\overline{U}_{i\alpha}(p^2 = m_i^2)$. Unfortunately, this definition also shows gauge dependence. From Eq. (20) we obtain

$$\partial_\xi [\delta \bar{u}_{ij}^L(m_i^2)] = \frac{1}{2} \left(m_j \Lambda_{Lij} + m_i \overline{\Lambda}_{Lij} + 2 m_i \Lambda_{Rij} + \Lambda_{DRij} - \overline{\Lambda}_{DLij} \right) (p^2 = m_i^2). \tag{31}$$

In Eq. (31) the factor $1/(m_i^2 - m_j^2)$ is again cancelled. To avoid this gauge dependence, the gauge-dependent term of the self energy (7) has to be rearranged such that it vanishes in the counterterm (30). As the second example, Ref. [7] proposed to renormalize the CKM matrix in terms of the zero-momentum self energies for quarks. The counterterm is then

$$\delta u_{ij}^L([7]) = \frac{1}{m_i^2 - m_j^2} \left[\frac{1}{2}(m_i^2 + m_j^2)\Sigma_{Lij}(0) + m_i \Sigma_{DLij}(0) + m_j \Sigma_{DRij}(0) \right] . \tag{32}$$

This definition gives the renormalized CKM matrix which is gauge-independent and UV finite. However, its validity relies on the fact that quark couplings to $W^\pm$ are purely left-handed. Moreover, Eq. (32) does not absorb the $\mathcal{O}(1/(m_i^2 - m_j^2))$ singularity. Thus, this definition has to be greatly modified for the renormalization of other mixing matrices.

4 CKM matrix: example for fermion mixing

In this and the next sections we show the explicit form of the gauge dependence of the on-shell mixing matrices, both for fermions and for scalars. In this section we discuss the on-shell CKM matrix, following previous studies [7, 8].

The off-diagonal parts of the one-loop self energies $\Sigma^q_{ij}(p)$ of the quarks receive gauge-dependent contribution from the $W^\pm$ loops [3, 4]. The ξ_W dependent part of $\Sigma^u_{ij}(p)$ for up-type quarks $u_i = (u, c, t)$, namely the difference from the result in the $\xi_W = 1$ gauge, takes the following form;

$$\begin{aligned}
\Sigma^u_{ij}(p)|_{\xi_W} &= (1-\xi_W)\frac{g_2^2}{32\pi^2}\sum_k V_{ik}V^*_{jk}\left\{(\not{p}-m_{u_i})\beta^{(1)}_{Wd_k}(p^2)\not{p}P_R(\not{p}-m_{u_j})\right. \\
&\quad -(\not{p}-m_{u_i})P_L\left[m^2_{d_k}\beta^{(0)}_{Wd_k}(p^2)-m_{u_j}\beta^{(1)}_{Wd_k}(p^2)\not{p}+\frac{1}{2}\alpha_W\right] \\
&\quad \left.-\left[m^2_{d_k}\beta^{(0)}_{Wd_k}(p^2)-m_{u_i}\beta^{(1)}_{Wd_k}(p^2)\not{p}+\frac{1}{2}\alpha_W\right]P_R(\not{p}-m_{u_j})\right\}. \qquad (33)
\end{aligned}$$

Here we define

$$\frac{i}{16\pi^2}\alpha_i = \int\frac{d^nq}{(2\pi)^n}\frac{1}{(q^2-m_i^2)(q^2-\xi_i m_i^2)}, \qquad (34)$$

$$\frac{i}{16\pi^2}\beta^{(0)}_{ij}(p^2) = \int\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2-m_i^2)(q^2-\xi_i m_i^2)((q+p)^2-m_j^2)}, \qquad (35)$$

$$\frac{i}{16\pi^2}\beta^{(1)}_{ij}(p^2)p_\mu = \int\frac{d^4q}{(2\pi)^4}\frac{(q+p)_\mu}{(q^2-m_i^2)(q^2-\xi_i m_i^2)((q+p)^2-m_j^2)}, \qquad (36)$$

where $n = 4-2\epsilon$. $\Sigma^d_{ij}(p)|_{\xi_W}$ for down-type quarks $d_i = (d, s, b)$ is obtained by replacing (u_i, u_j, d_k, $V_{ik}V^*_{jk}$) in Eq. (33) by (d_i, d_j, u_k, $V^*_{ki}V_{kj}$). Equation (33) is equivalent to the results in Refs. [8, 16], except that Eq. (33) includes the gauge-dependent part of the Higgs VEV shift in Σ^u_{ii}, by tadpoles with $W^\pm$ and associated unphysical modes. This corresponds to defining renormalized Higgs VEV as the minimum of the tree-level potential [23, 24, 16], which is gauge-independent in the $\overline{\mathrm{MS}}$ scheme. By the addition of the Higgs VEV shift, Eq. (33) manifestly satisfies the Nielsen identity (18). Instead, one may also add the counterterms for pole masses of quarks to the diagonal elements to satisfy Eq. (18). This difference does not affect the present discussion.

The counterterm for the on-shell CKM matrix in the original definition [4], without separating Eq. (33), has gauge dependence as

$$(\delta V_{ij})_\xi = X^u_{ik}V_{kj} + V_{il}X^d_{jl}. \qquad (37)$$

X^u_{ik} $(i \neq k)$ is obtained from Eq. (33) as

$$\begin{aligned}
X^u_{ik} &= (1-\xi_W)\frac{g_2^2}{64\pi^2}V_{il}V^*_{kl}\left[-m^2_{u_k}\beta^{(1)}_{Wd_l}(m^2_{u_k})+m^2_{u_i}\beta^{(1)}_{Wd_l}(m^2_{u_i})\right. \\
&\quad \left.+m^2_{d_l}\beta^{(0)}_{Wd_l}(m^2_{u_k})-m^2_{d_l}\beta^{(0)}_{Wd_l}(m^2_{u_i})\right]. \qquad (38)
\end{aligned}$$

X^d_{jl} has a similar form. Equation (37) causes gauge-dependent amplitudes for the $W\bar{u}_i d_j$ interactions [7, 8, 9]. Numerically, Eq. (37) is greatly suppressed, partly by the Glashow-Iliopoulos-Maiani (GIM) mechanism [25], and completely negligible in practice [4]. The relative corrections are largest to (V_{cb}, V_{ub}, V_{td}, V_{ts}), but are at most $\mathcal{O}(10^{-6})$. Nevertheless, this is not satisfactory for theoretical point of view. The study in previous section shows, however, that one can give the counterterm δV in terms of $\xi = 1$ parts of Σ^u_{ij} and Σ^d_{ij}. The original calculation in Ref. [4] is thus interpreted as a gauge-independent one after the rearrangement.

5 Left-right mixing of squarks: example for scalar mixing

We next consider the renormalization of the left-right mixing of squarks in the MSSM, for an example for the mixing of scalar particles. For simplicity, we treat the mixing of two eigenstates of the top squarks, ignoring CP violation and mixing with different generations.

The gauge eigenstates ($\tilde{q}_L$, $\tilde{q}_R$) of squarks, which are the superpartners of a quark q, mix with each other by spontaneous breaking of SU(2)×U(1) gauge symmetry [1, 2]. Their mass eigenstates $\tilde{q}_i (i = 1, 2)$ are related to the gauge eigenstates $\tilde{q}_\alpha (\alpha = L, R)$ by $\tilde{q}_i = R^{\tilde{q}}_{i\alpha} \tilde{q}_\alpha$ with the left-right mixing matrix

$$R^{\tilde{q}}_{i\alpha} = \begin{pmatrix} \cos\theta_{\tilde{q}} & \sin\theta_{\tilde{q}} \\ -\sin\theta_{\tilde{q}} & \cos\theta_{\tilde{q}} \end{pmatrix} . \tag{39}$$

The renormalization of the squark sector is often performed by specifying the poles masses of ($\tilde{q}_1$, $\tilde{q}_2$) and the mixing angle $\theta_{\tilde{q}}$, as in Refs. [26, 27, 28, 29, 30, 31, 32, 6, 33, 34]. Following the result in section 2, the counterterm $\delta\theta_{\tilde{q}}$ is given by [6, 34]

$$\delta\theta_{\tilde{q}} = \delta r_{12} = \frac{1}{2(m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2})} \left[\Pi^{\tilde{q}}_{12}(m^2_{\tilde{q}_1}) + \Pi^{\tilde{q}}_{12}(m^2_{\tilde{q}_2}) \right] , \tag{40}$$

with $\Pi^{\tilde{q}}_{12}(p^2)$ the off-diagonal self energy of squarks in the tree-level mass basis. Although many other on-shell definitions [26, 27, 28, 29, 30, 31, 32] have been used in the studies of the SUSY QCD corrections, they are either unable to be applied for other loop corrections, or too specific for the squark processes considered there.

We consider the on-shell mixing matrix for top squarks $\tilde{t}_i$. The gauge-dependent part of the unrenormalized two-point function $\Pi^{\tilde{t}}_{ij}(q^2)$, namely the difference from the results in the $\xi_Z = \xi_W = 1$ gauge [35], takes the following form:

$$\begin{aligned} \Pi^{\tilde{t}}_{ij}(p^2)|_\xi \quad &= \quad \frac{g_Z^2}{16\pi^2}(1-\xi_Z) \sum_k (R^{\tilde{t}}_{i1} R^{\tilde{t}}_{k1} T_{3t} - \delta_{ik} s^2_W Q_t)(R^{\tilde{t}}_{k1} R^{\tilde{t}}_{j1} T_{3t} - \delta_{kj} s^2_W Q_t) \\ &\times \Big[-\frac{1}{2}(2p^2 - m^2_{\tilde{t}_i} - m^2_{\tilde{t}_j})\alpha_Z + \Big\{ (p^2 - m^2_{\tilde{t}_i})(p^2 - m^2_{\tilde{t}_j}) \end{aligned}$$

$$
\begin{aligned}
&+(p^2-m_{\tilde{t}_i}^2)(m_{\tilde{t}_j}^2-m_{\tilde{t}_k}^2)+(m_{\tilde{t}_i}^2-m_{\tilde{t}_k}^2)(p^2-m_{\tilde{t}_j}^2)\Big\}\,\beta^{(0)}_{Z\tilde{t}_k}(p^2)\Big]\\
&+\frac{g_2^2}{32\pi^2}(1-\xi_W)R^{\tilde{t}}_{i1}R^{\tilde{t}}_{j1}\left[-\frac{1}{2}(2p^2-m_{\tilde{t}_i}^2-m_{\tilde{t}_j}^2)\alpha_W\right.\\
&+\sum_k (R^{\tilde{b}}_{k1})^2\Big\{(p^2-m_{\tilde{t}_i}^2)(p^2-m_{\tilde{t}_j}^2)+(p^2-m_{\tilde{t}_i}^2)(m_{\tilde{t}_j}^2-m_{\tilde{b}_k}^2)\\
&\left.+(m_{\tilde{t}_i}^2-m_{\tilde{b}_k}^2)(p^2-m_{\tilde{t}_j}^2)\Big\}\,\beta^{(0)}_{W\tilde{b}_k}(p^2)\right]\,. \qquad (41)
\end{aligned}
$$

Here $T_{3t}=1/2$, $Q_t=2/3$, and $s_W^2=\sin^2\theta_W$. As before, Eq. (41) includes the gauge-dependent shifts of the two Higgs VEVs for gauge-independent renormalization of the VEVs. In contrast to the SM case, they also contribute to the $i\neq j$ parts. The result (41) satisfies the Nielsen identity (25).

The magnitude of the gauge dependence of the on-shell $\delta\theta_{\tilde{t}}$ is very sensitive to squark parameters. For a parameter choice ($M_{\tilde{Q}}$, $M_{\tilde{U}}$, $M_{\tilde{D}}$)=(350, 300, 400) GeV, $\tan\beta=4$, (μ, A_t, A_b)=(-400, 300, 0) GeV, and $0<\xi<10$, ξ_W and ξ_Z dependent parts of $\delta\theta_{\tilde{t}}$ may be as large as 0.008 and 0.003, respectively. Although too small for realistic phenomenology, they are much larger than the ξ_W dependence of the on-shell CKM matrix. This is partly due to the absence of the GIM cancellation, following from that $\tilde{t}_L$ and $\tilde{t}_R$ have different gauge representations. As is already shown, these gauge dependence of $\theta_{\tilde{t}}$ can be avoided by removing the contribution of Eq. (41) from off-diagonal wave function corrections δZ_{12} for top squarks, cancelling it by other gauge dependences of the amplitude, and then giving $\delta\theta_{\tilde{t}}$ by the remaining part of δZ_{12}.

6 Conclusion

In this paper, we investigated the gauge parameter dependence of the on-shell renormalized mixing matrices for scalars and fermions at the one-loop level. It has been shown recently that the on-shell renormalization of the CKM matrix in the definition by Ref. [4] is gauge dependent. By using the Nielsen identities for self energies, we demonstrated that this gauge dependence exists for the on-shell mixing matrices in general cases. We also showed that this gauge dependence can be avoided by the following procedure; split the gauge-dependent parts from the off-diagonal wave function corrections in the manner similar to the pinch technique, and then giving the counterterm for the mixing matix in terms of the remaining, gauge-independent parts. The subtraction of the UV divergence and $\mathcal{O}(1/(m_i^2-m_j^2))$ singularity is not affected by this modification. The on-shell scheme in Ref. [4] in the $\xi=1$ gauge can be then regarded as gauge-independent one. Finally, we presented explicit calculation of the gauge-dependence of the mixing matrices in two cases, CKM matrix and left-right mixing of squarks, and verified the result from the Nielsen identities.

We did not treat the mixings of the gauge bosons and of the Higgs bosons. In principle, our method would also be applicable for these mixings. When applied for the mixing of the gauge bosons γ and Z, the square of the renormalized mixing angle $\sin^2\theta_W(\mathrm{OS})$ agrees with the effective angle $s_*^2(m_Z^2)$ defined in Ref. [36], at the one-loop level. But the inclusion of the absorptive part of the Z boson propagator is necessary for realistic studies. The correction to the mixing of the MSSM Higgs bosons in diagrammatic calculation [37, 38, 39] is a very interesting subject. However, due to the mixing of physical Higgs bosons with unphysical modes, a separate consideration is necessary. We expect to study the case of the MSSM Higgs bosons in future.

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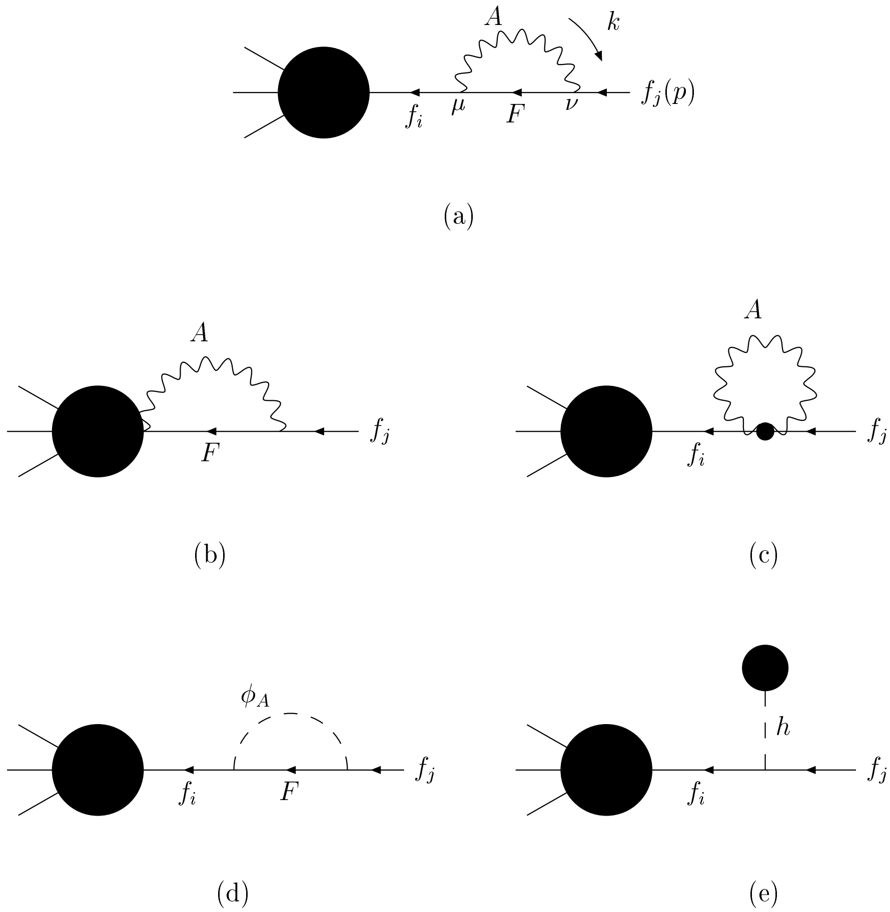

Figure 1: The gauge-dependent contributions to δZ_{ij} for a general process with external on-shell f_j, which is either a fermion or a scalar, from the loops of massive gauge boson A and intermediate particle F. Graphs (b, c) are the "pinch terms" stemming from (a). Graph (d) is a contribution of the NG boson ϕ_A associated with A. Graph (e) represents the shift of the VEV of Higgs bosons h by the loops of A, ϕ_A, and Fadeev-Popov ghosts. Inclusion of (e) is necessary for gauge-independent renormalization of the Higgs VEVs.